Notes

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November 10, 2011

1 Week 7

1.1 Hypothesis Testing

Recall, last week we *briefly* worked through an example using Confidence Intervals. This technique will work in all cases here as well, but we are going to work to develop a more general framework.
There broadly, there are "two-ways":

- P-value Approach
- Critical Value Approach

Steps

- Set α level
- State Null and Alternative Hypothesis
- Calculate a Test Statistic (**z** or sometimes **t**)
- Check p-value
- Draw Conclusion

1.2 Proportion

Test statistic: $z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ Confidence Interval: Estimate $\pm$ Margin of Error

- Margin of Error = Standard Deviation $*$ Z-score
- $\hat{p} \pm 1.96 * \sqrt{\frac{p_0(1-p_0)}{n}}$

Example

- Set $\alpha = .05 \rightarrow z = 1.96$
- State Null and Alternative Hypothesis:
  - H_0: The coin is fair, p = .50
  - H_A: The coin is unfair, p $\neq$ .50

- Calculate Test Statistic
  - $z = \frac{\hat{p}-p_0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} = \frac{0.55-0.5}{\sqrt{\frac{0.55(1-0.55)}{100}}} = \frac{0.05}{\frac{.55*.45}{100}} = 1.005$
- Get P-value from a normal table
- Conclude, because this is fairly likely, we do not reject the null hypothesis.

1.3 Two-Samples

Remember, this is the case where we need to calculate the pooled probabilities.

Is the proportion of people who believe H_0: $p_1 = p_2$
H_A: $p_1 \neq p_2$

Test-Statistic : $t_{df} = \frac{\hat{p}_m - \hat{p}_w}{\sqrt{\frac{pp(1-pp)}{n_m} + \frac{pp(1-pp)}{n_w}}}$, where $\hat{p}_p = \frac{k_m + k_w}{n_m + n_w}$

To Calculate the 95% Confidence Interval:

$CI = \hat{p_1} - \hat{p_2} \pm ME$, where

$ME = 1.96 \pm \sqrt{\frac{\hat{p_1}(1-\hat{p_1})}{n_1} + \frac{\hat{p_2}(1-\hat{p_2})}{n_2}}$

1.4 One Sample Mean

To be clear, there is a difference between $SE(x)$ & $SE(\bar{x})$.

- $s = SE(x) = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$ = Standard Deviation of the Sample
- $SE(\bar{x}) = \sqrt{\frac{s^2}{n}}$ = Standard Error of the Sample Mean

Pushing Forward....

- Confidence Interval: $CI = \bar{x} \pm SE_{\bar{x}} * t_{\alpha,n-1}$
- Test Statistic for One Sample: $t_{\alpha,n-1} = \frac{\bar{x}-\mu_0}{SE(\bar{x})}$

Again, Pushing Forward...

1.5 Two Sample Means

- Confidence Interval: $CI = \bar{x_1} - \bar{x_2} \pm SE_{\bar{x_1}-\bar{x_2}} * t_{\alpha,n-1}$
- $SE_{\bar{x_1}-\bar{x_2}} = \sqrt{\frac{s_1^2}{n1} + \frac{s_2^2}{n_2}}$
- $t_{\alpha,df} = \frac{\bar{x_1}-\bar{x_2}}{SE_{\bar{x_1}-\bar{x_2}}}$

The astute test taker will notice that there was a df term in the last equation, to calculate the number of df on has, follow the following, convoluted schema:

- Equal Sample Size, Equal Variance? $\rightarrow df = 2n - 2$
- Unequal Sample Size, Equal Variance? $\rightarrow df = n_1 + n_2 - 2$
- Unequal Sample Size, Unequal Variance? Oh no... Buuut... $\rightarrow$
  $df = \frac{(\frac{s_1^2}{n_1} + s_2^2)^2}{\frac{(\frac{s_1^2}{n_1})^2}{(n_1 - 1)} + \frac{(\frac{s_2^2}{n_2})^2}{(n_2 - 1)}}$

1.6 Test for Independence of Two Categorical Variables

This seems likely to be on the exam; better know it.

Table 1: Votes

	PAN	PRI	PRD	
Employed	320	245	288	853
Student	98	24	17	139
Unemployed	18	19	5	42
Retired	17	2	2	21
	453	290	312	1055

To calculate the expected values, multiply the expected percentages $\times$ the row totals. That is, $Exp_{i,j} = \frac{c_j \times r_i}{n}$

Table 2: Calculation for Exp Values

	PAN	PRI	PRD	*sum*
Employed	$\frac{853\times453}{1055}$	$\frac{853\times290}{1055}$	$\frac{853\times312}{1055}$	853
Student	$\frac{139\times453}{1055}$	$\frac{139\times24}{1055}$	$\frac{139\times290}{1055}$	139
Unemployed	$\frac{42\times18}{1055}$	$\frac{42\times19}{1055}$	$\frac{42\times5}{1055}$	42
Retired	$\frac{21\times17}{1055}$	$\frac{21\times2}{1055}$	$\frac{21\times2}{1055}$	21
sum	453	290	312	1055

Table 3: Expected Values

	PAN	PRI	PRD
Employed	366	234	252
Student	59	38	41
Unemployed	18	11	12
Retired	9	5	6

Now, to calculate the test statistic, $\chi^2 = \sum \frac{(Obs - Exp)^2}{Exp} = 77.03$. Use this statistic, with a χ^2 table to calculate the p-value.

1.7 Difference Between Variances

- $H_0 : \sigma_1^2 = \sigma_2^2$
- $H_A : \sigma_1^2 \neq \sigma_2^2$
- $\frac{s_1^2}{s_2^2} \sim F_{n_1,n_2}$

1.8 Discussion of α and β

- Type I Error: Reject the Null, when in fact we should accept it. "False Positive"
- Type II Error: Fail to Reject the Null when in fact we should reject it. "False Negative"
- α & β have inverse relationships, that is, the greater the statistical power, β, the lower the probability of Type II Error. Typically, the only way to increase statistical power is to increase the sample size. The narrower the α value desired, the higher the statistical power must be.

	Null Hypothesis (H_0) is true	Alternative Hypothesis (H_1) is true
Fail to Reject Null Hypothesis	Right decision	Wrong decision Type II Error False Negative
Reject Null Hypothesis	Wrong decision Type I Error False Positive	Right decision

Figure 1: Relationship between α, β and Type-I and Type-II error.